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PERFORMANCE ANALYSIS OF HIGH-ENERGY CHEMICAL STAGES FOR INTERPLANETARY MISSIONS.

PART I: DEPARTURE FROM EARTH ORBIT

By

Walter H. Stafford and Carmen R. Catalfamo

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ABSTRACT 22018

The effect of thrust-to-weight ratios and specific impulses on trajectory parameters has been investigated for hyperbolic escape from an Earth orbit. The initial thrust vector was applied tangentially in the direction of motion of the velocity vector of an orbit with a radius of 6556 km. Specific impulses of 400 to 500 sec and thrust-to-weight ratios of 0.2 to 1.0 were used.

The results of the study are presented graphically.

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FLIGHT OPERATIONS SECTION
ADVANCED FLIGHT SYSTEMS BRANCH
PROPULSION AND VEHICLE ENGINEERING DIVISION

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DEFINITION OF SYMBOLS

Symbol	Definition
F	Thrust, kp
g	Gravitational acceleration, m/sec^2
h	Altitude, km
Δh	Altitude change, $h - h_o$, km
I_{sp}	Specific impulse, sec
m	Mass, $\frac{kp - sec^2}{m}$
r	Radius, km
t	Time, sec
Δt	Incremental time, sec
V	Velocity
V^*	Comparative velocity
ΔV	Characteristic velocity
V_∞	Hyperbolic excess velocity
W_o	Gross weight, kp
F/W_o	Initial thrust-to-weight ratio (based on weight at earth sea level)
W_A	Stage weight, $W_o - W_L$
W_L	Payload weight, kp
W_P	Propellant weight, kp
f	Stage mass fraction, W_P/W_A
H	Energy
ζ	Propellant mass fraction, W_P/W_o
ϑ	Flight path angle from vertical, deg
μ	Gravitational constant for the Earth, 398,606.6 km^3/sec^2
ψ	Central angle, deg

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
X	Range, km

Subscripts

P	Propellant
C	Burnout
o	Initial
id	Ideal
f	Final
ex	Exhaust
$\oplus$	Earth

Abbreviations

km	Kilometer
m	Meter
sec	Second
kp	Kilopond

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SUMMARY

The effect of thrust-to-weight ratios and specific impulses on trajectory parameters has been investigated for hyperbolic escape from an Earth orbit. The initial thrust vector was applied tangentially in the direction of motion of the velocity vector of an orbit with a radius of 6556 km. Specific impulses of 400 sec to 500 sec and thrust-to-weight ratios of 0.2 to 1.0 were used.

The results of the study are presented graphically.

SECTION I. INTRODUCTION

Of fundamental importance in planning interplanetary round-trip-missions is a study of the trajectory requirements. The sizing of boost vehicles is dependent, to a large extent, on the velocity requirements of the particular trajectory chosen.

The purpose of this study is to present a method for determining the trajectory parameters for a specific mission when the hyperbolic excess velocity is known.

The approach used was to determine the trajectory parameters at burnout, convert the characteristic velocity to a hyperbolic excess velocity, and then present the data graphically. This scheme was used for several specific impulse values and thrust-to-weight ratios. The equations of motion were integrated on a RECOMP II computer, using a Runge-Kutta numerical integration procedure.

SECTION II. ASSUMPTIONS

The following is a summary of the basic assumptions used in this study:

1. Acceleration of a single stage from a reference orbit about the earth, using a constant thrust directed along the velocity vector.

2. Reference orbit was circular with a radius of 6556 km.

3. Constant specific impulse values:

   a. 400 sec

   b. 425 sec

   c. 450 sec

   d. 475 sec

   e. 500 sec

4. The thrust-to-weight ratio for a chemical stage was varied parametrically from 0.2 to 1.0.

5. Mean spherical earth:

   $\mu = 398,606.6 \text{ km}^3/\text{sec}^2$

   $r = 6371.27 \text{ km}$

SECTION III. ANALYSIS

For interplanetary mission programs, it is assumed that one mode of flight will be by way of transfer from a circular orbit around the Earth.

For interplanetary flight the ideal[1] total energy that must be imparted to the spacecraft is the ideal energy required to escape the gravitational field of the planet plus the energy required to change its path about the Sun. The ideal energy required to escape the gravitational attraction of a planet can be determined from two-body mechanics to be $H_{esc} = 2\mu/r$ and the energy needed to alter the flight path about the Sun, H_{∞}, is determined by characteristics of the interplanetary trajectory.

For determining vehicle size necessary to inject the spacecraft into the interplanetary trajectory, it is convenient to express the ideal total energy, $H = (2\mu/r) + H_{\infty}$, in terms of a burnout velocity. This produces equations of the following forms:

$$V_C = \sqrt{\frac{2\mu}{r} + H_{\infty}}$$

or

$$V_C = \sqrt{(V_{esc})^2 + (V_{\infty})^2}$$

When considering finite vehicle systems there is an additional energy requirement, H_{loss}, which is due to expending the propellants at different energy levels. Therefore, the total velocity increment for the injecting stage is now

$$\Delta V = V_C - V_o + V_{loss}$$

where V_o is the initial velocity. The vehicle mass characteristics can be determined from the equation

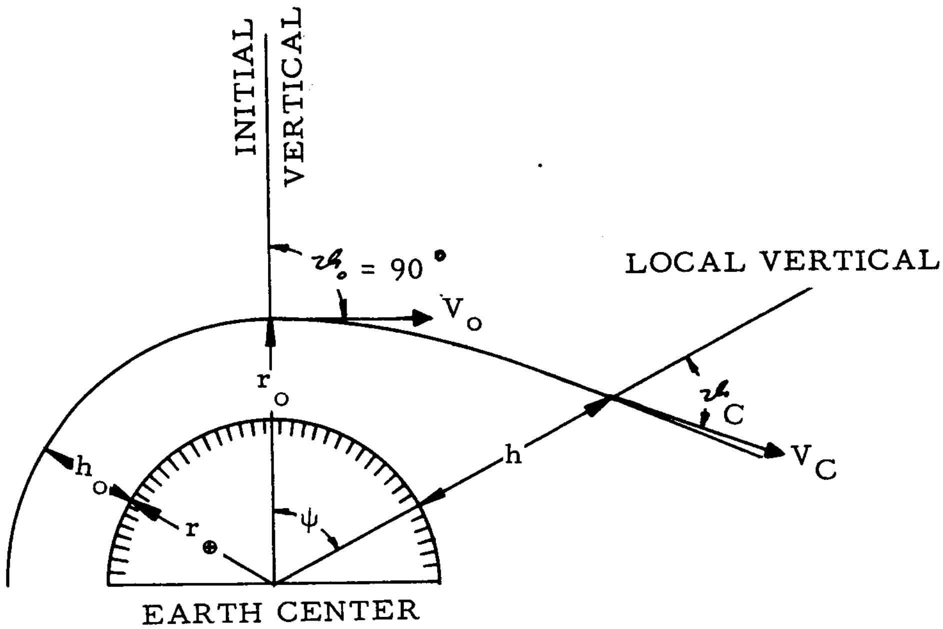
$$\frac{W_o}{W_C} = e^{\frac{\Delta V}{V_{ex}}}$$

The purpose of this study was to determine the effects of various initial vehicle thrust-to-weight ratios and system specific impulses for given hyperbolic excess velocities, on the mass characteristics of the vehicle. To accomplish this, the two-degree-of-freedom equations of motion were numerically integrated for a vehicle leaving a reference orbit about the Earth and burning to an injection with a specified

---

(1) The term "ideal" refers to an instantaneous change of energy.

hyperbolic excess velocity. For reference, a circular orbit with a radius of 6556 km and a velocity of 7798 m/sec was taken. The thrust vector was applied tangentially in the direction of motion of the velocity vector. The parameters thus obtained at injection were plotted.



Referring to the sketch above, computations were made for a point mass moving in a plane using the following equations of motion:

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{\mu_\oplus}{r^2} \cos \vartheta \tag{1}$$

$$V \dot{\vartheta} = \frac{F \sin \alpha}{m} + \left(\frac{\mu_\oplus}{r^2} - \frac{V^2}{r} \right) \sin \vartheta \tag{2}$$

$$\dot{r} = V \cos \vartheta \tag{3}$$

$$\dot{\psi} = \frac{V \sin \vartheta}{r} \tag{4}$$

where

$$m = m_o + \int \dot{m}\, dt \tag{5}$$

and

$$\dot{m} = -\frac{F}{V_{ex}} \tag{6}$$

The velocity and flight path angle may be obtained by integrating the equations of motion

$$V = \int \dot{V}\, dt \tag{7}$$

$$\vartheta = \int \dot{\vartheta}\, dt \tag{8}$$

The range and altitude can then be calculated by the relations

$$X = X_o + \int \frac{r_\oplus}{r}\, v \sin\vartheta\, dt \tag{9}$$

$$h = h_o + \int \dot{r}\, dt \tag{10}$$

and the central angle is

$$\psi = \int \frac{\dot{X}}{r_\oplus}\, dt \tag{11}$$

The initial weight of the vehicle is

$$W_o = W_C + W_P \tag{12}$$

SECTION IV. DISCUSSION OF RESULTS

The results of this investigation are shown in Figures 1 through 13. The characteristic velocity, ΔV, is plotted versus hyperbolic excess velocity with thrust-to-weight ratios as a parameter in Figures 1 through 5.

Figures 6 and 7 show the velocity losses due to gravity for specific impulse values of 400 sec and 500 sec respectively. It should be noted that these losses tend to zero as the thrust-to-weight ratio is increased. The flight path angle at injection is shown in Figure 8.

Figure 9 shows the change in altitude. This change is the difference between the reference orbit altitude and the altitude at injection into the interplanetary transfer trajectory. The change in other trajectory variables is shown in Figures 10 and 11. The vehicle mass characteristics can be determined from Figures 12 and 13.

SECTION V. CONCLUSIONS

From this parametric analysis, sufficient data is presented to enable the designer to make a preliminary design of an orbit launched interplanetary stage when the mission requirements are defined.

SECTION VI. GRAPHIC PRESENTATION

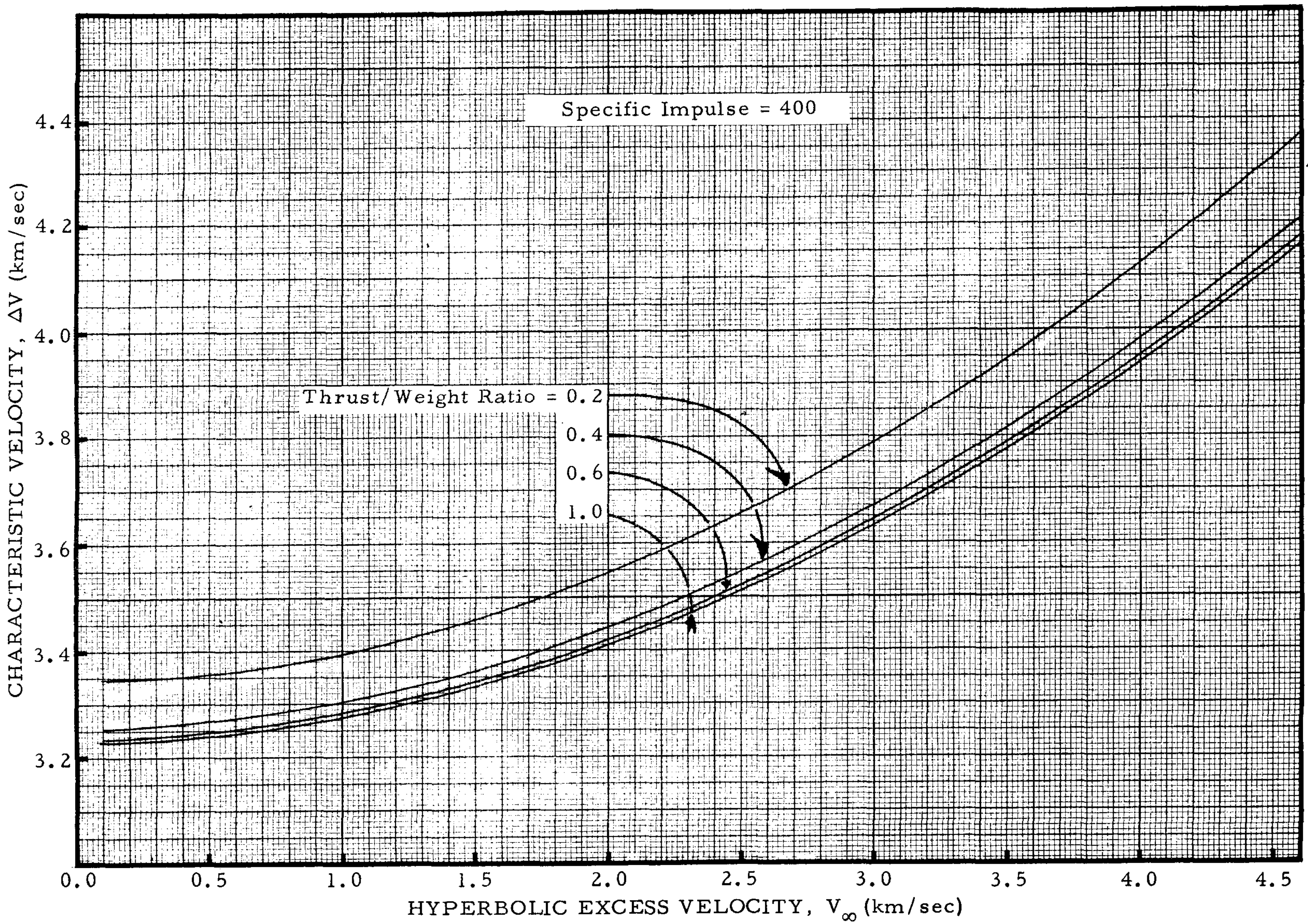


FIGURE 1a. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

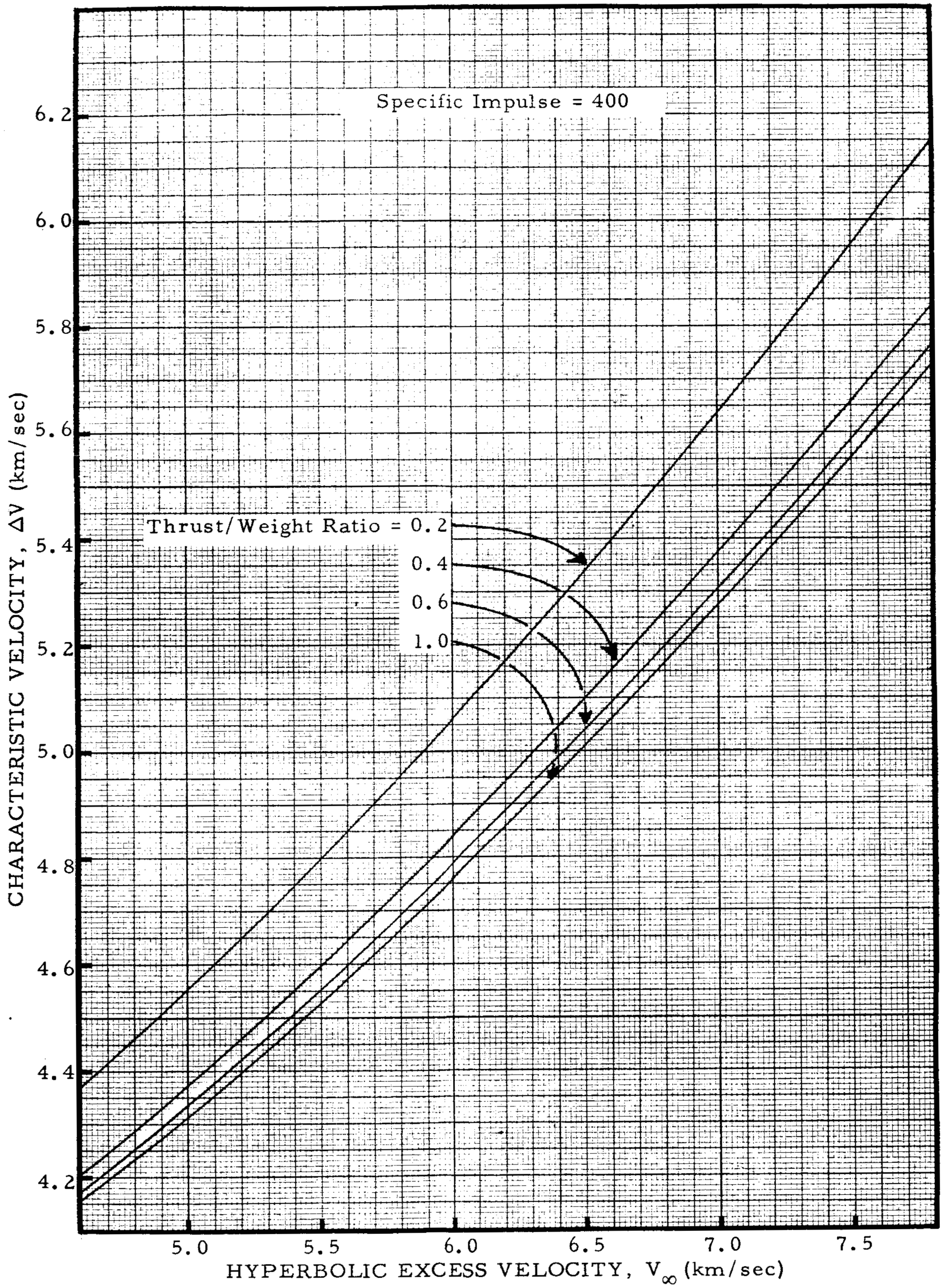


FIGURE 1b. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_{∞} (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

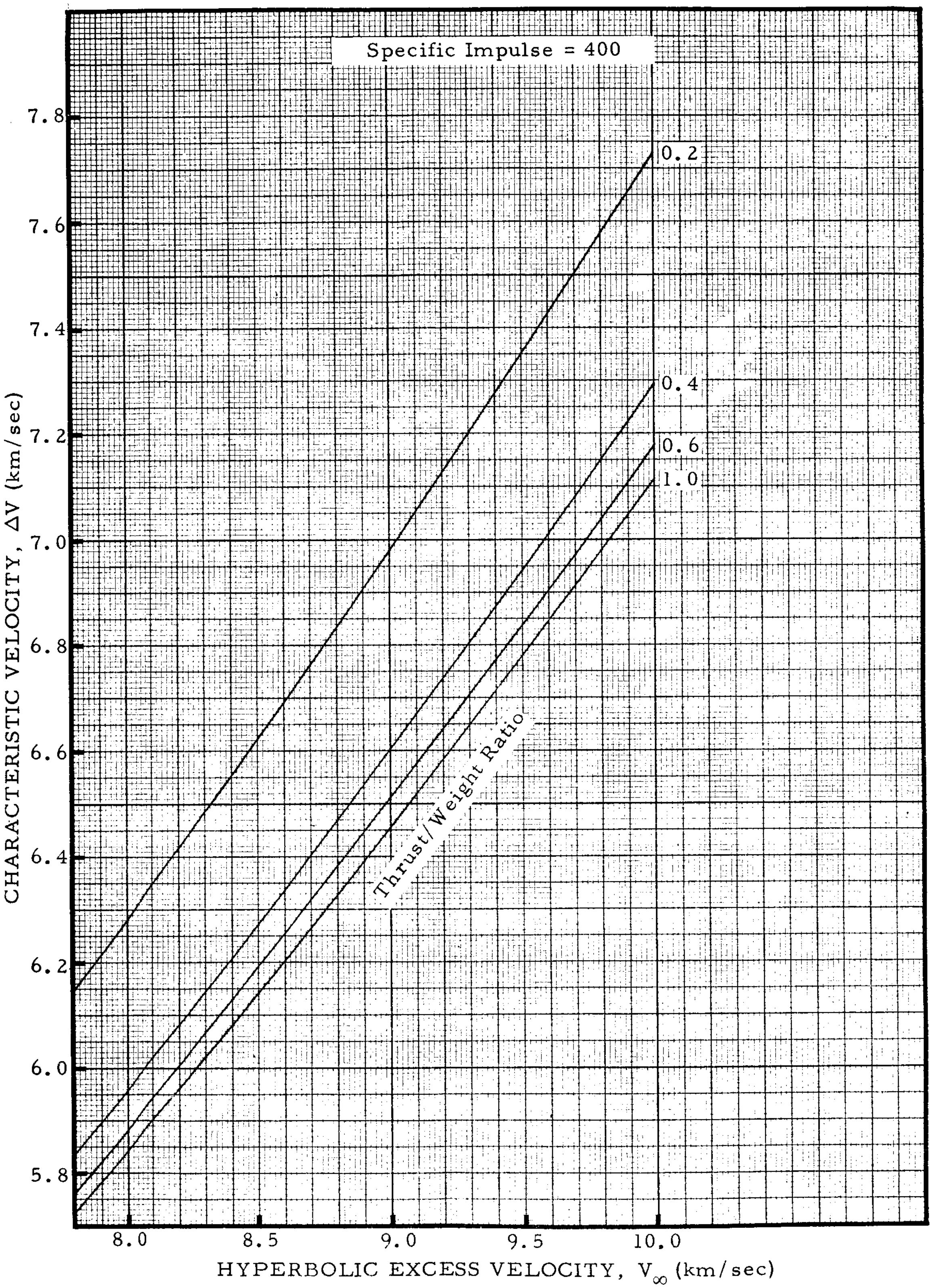


FIGURE 1c. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_{∞} (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

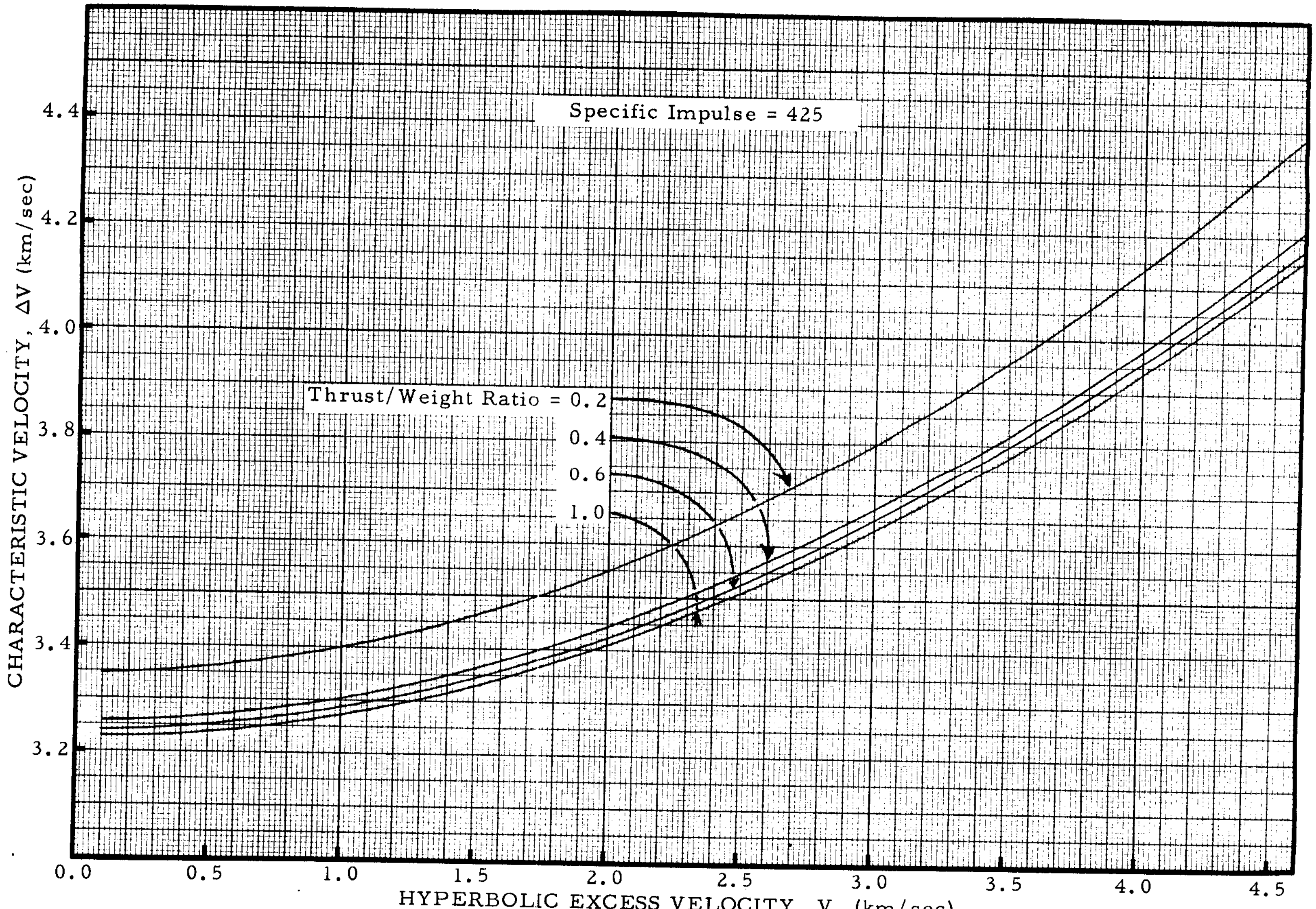


FIGURE 2a. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

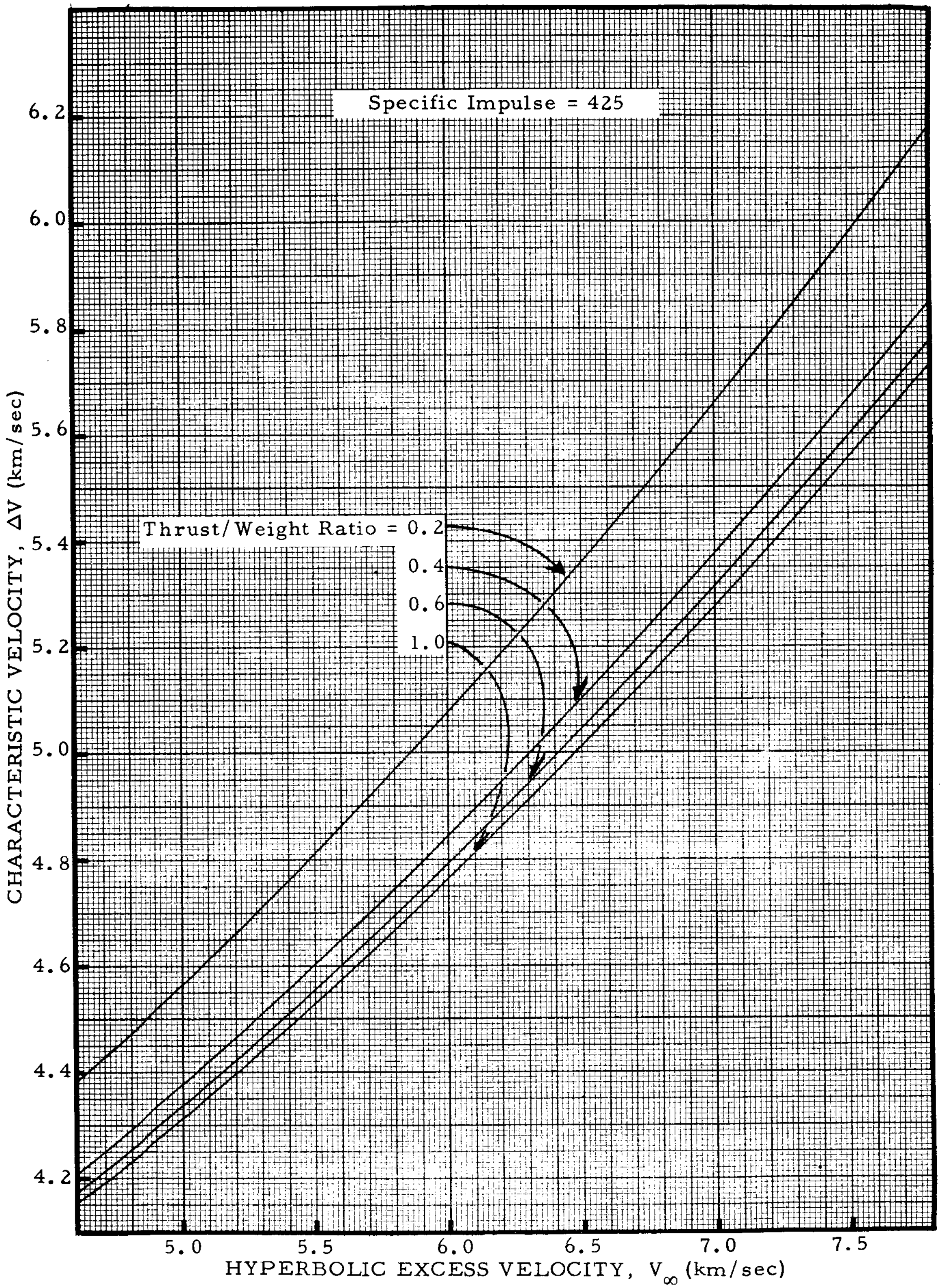


FIGURE 2b. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_{∞} (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

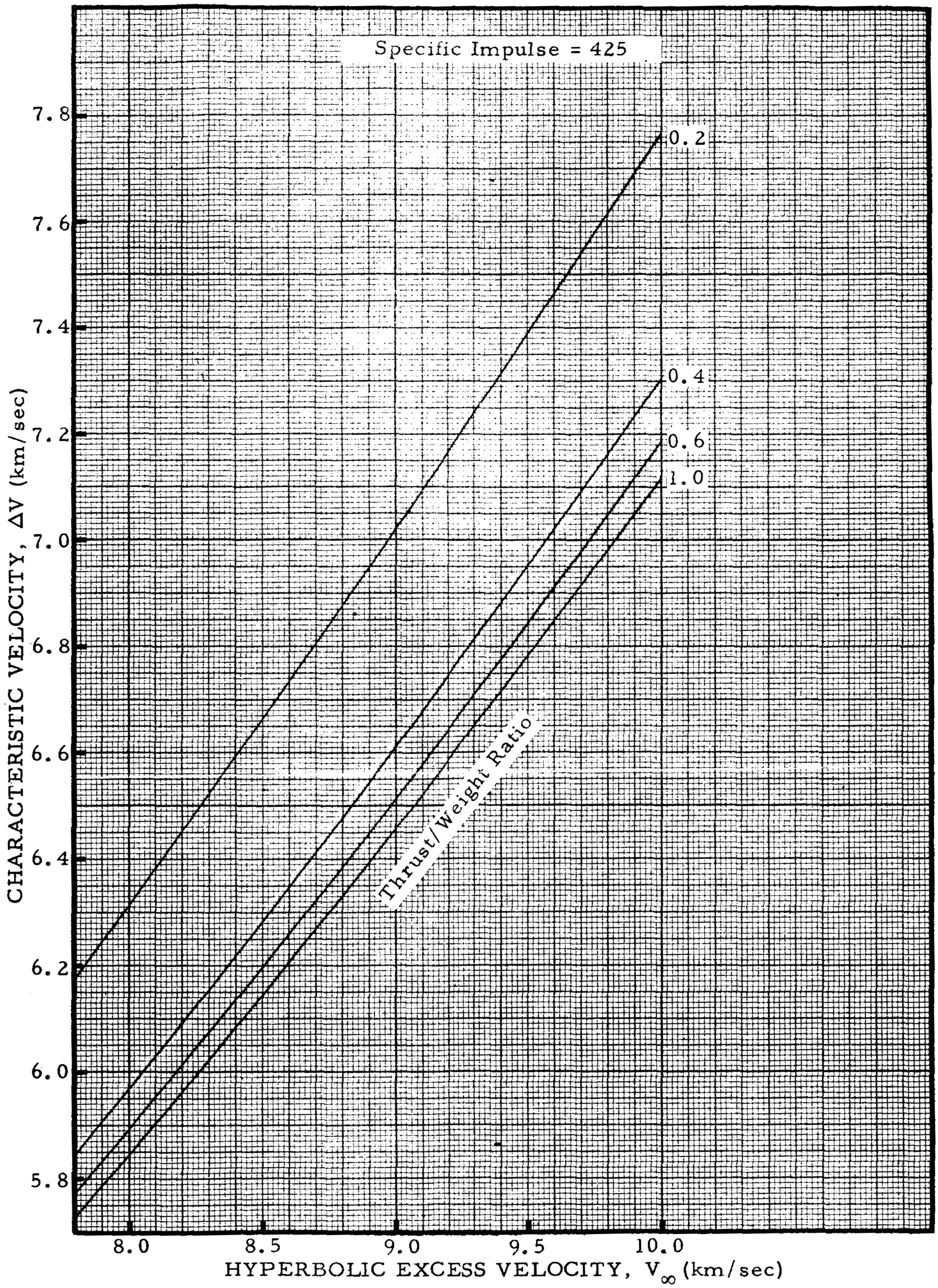


FIGURE 2c. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 425 SECONDS

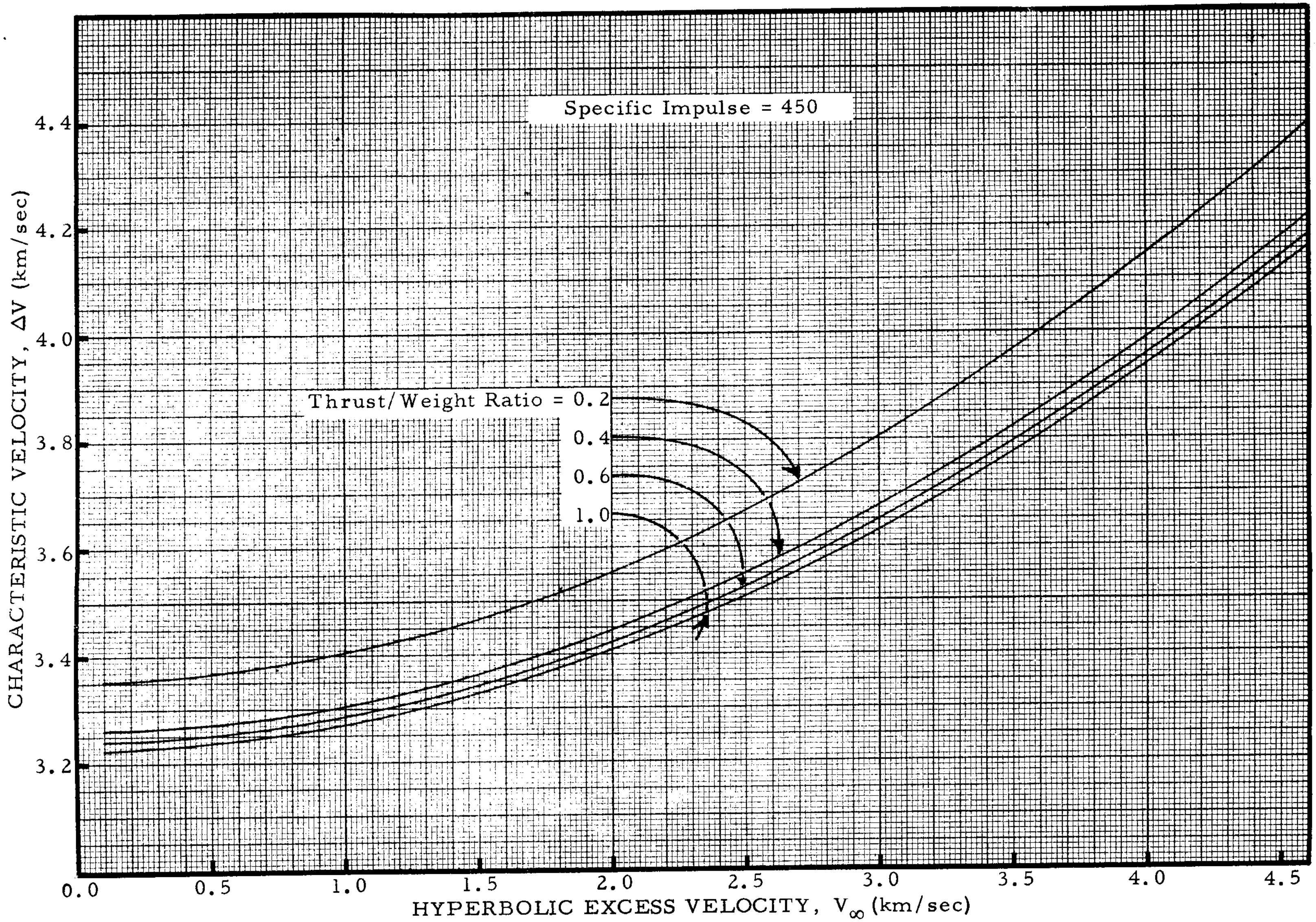


FIGURE 3a. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS

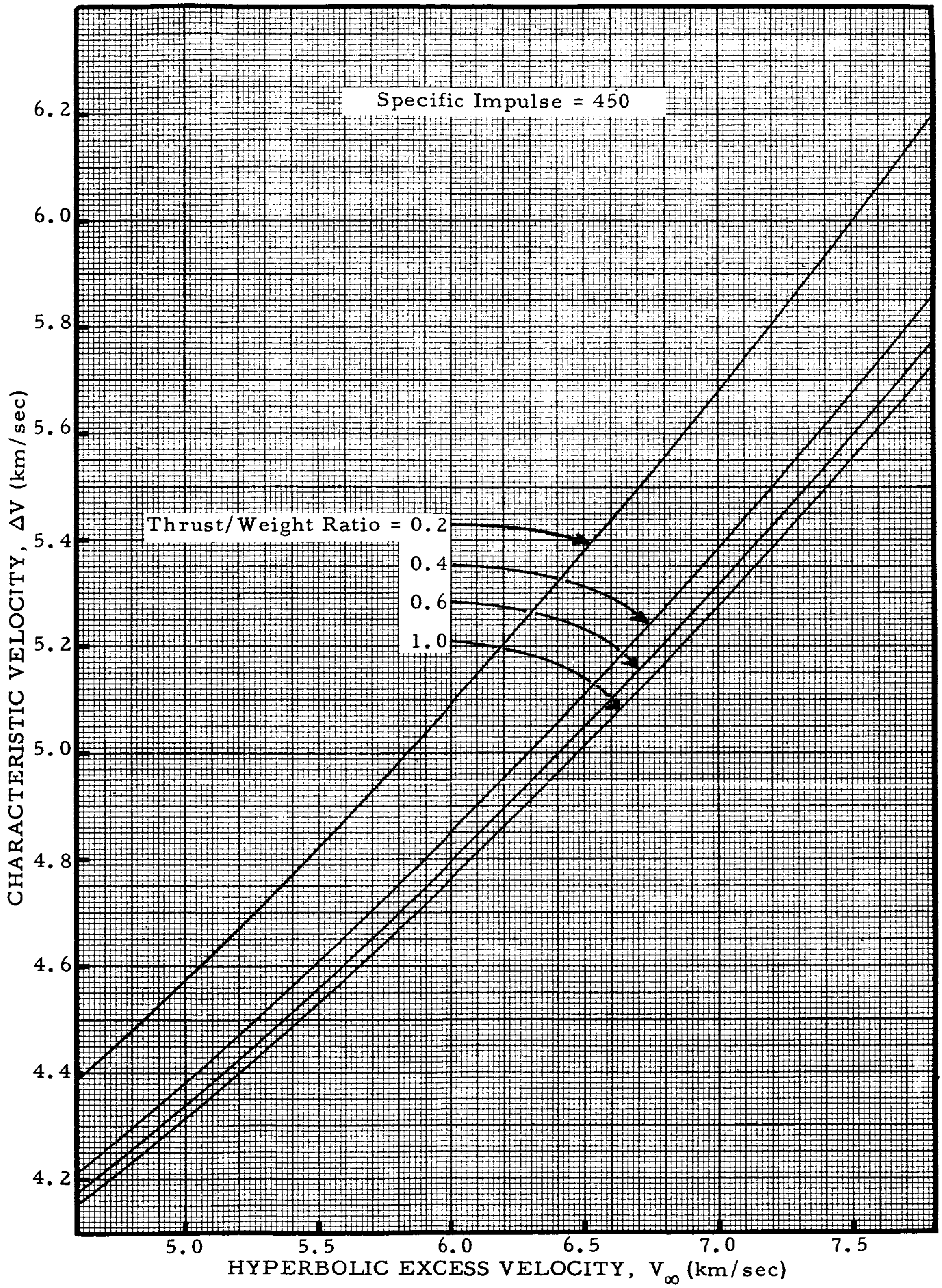
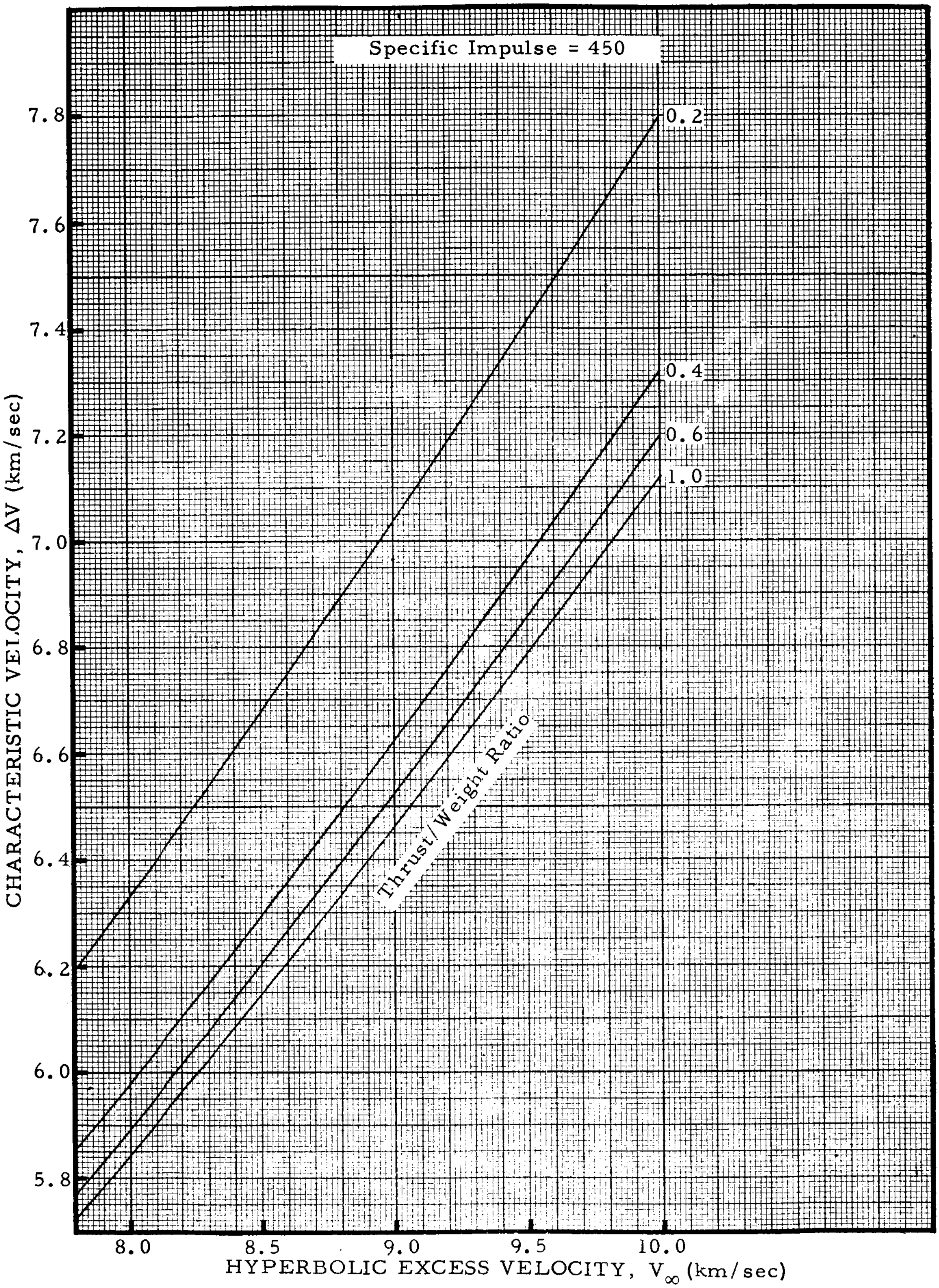


FIGURE 3b. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 450 SECONDS



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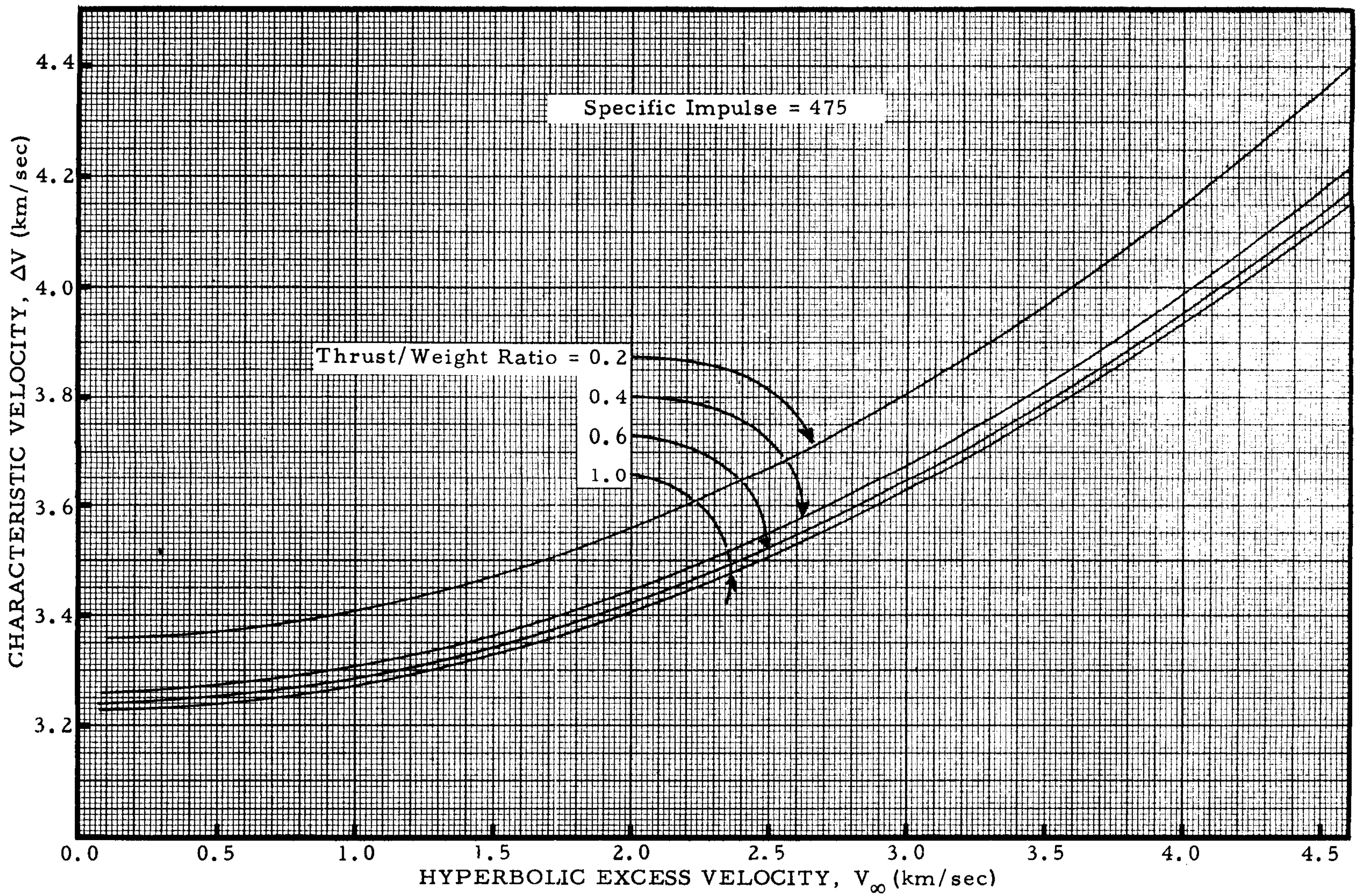


FIGURE 4a. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

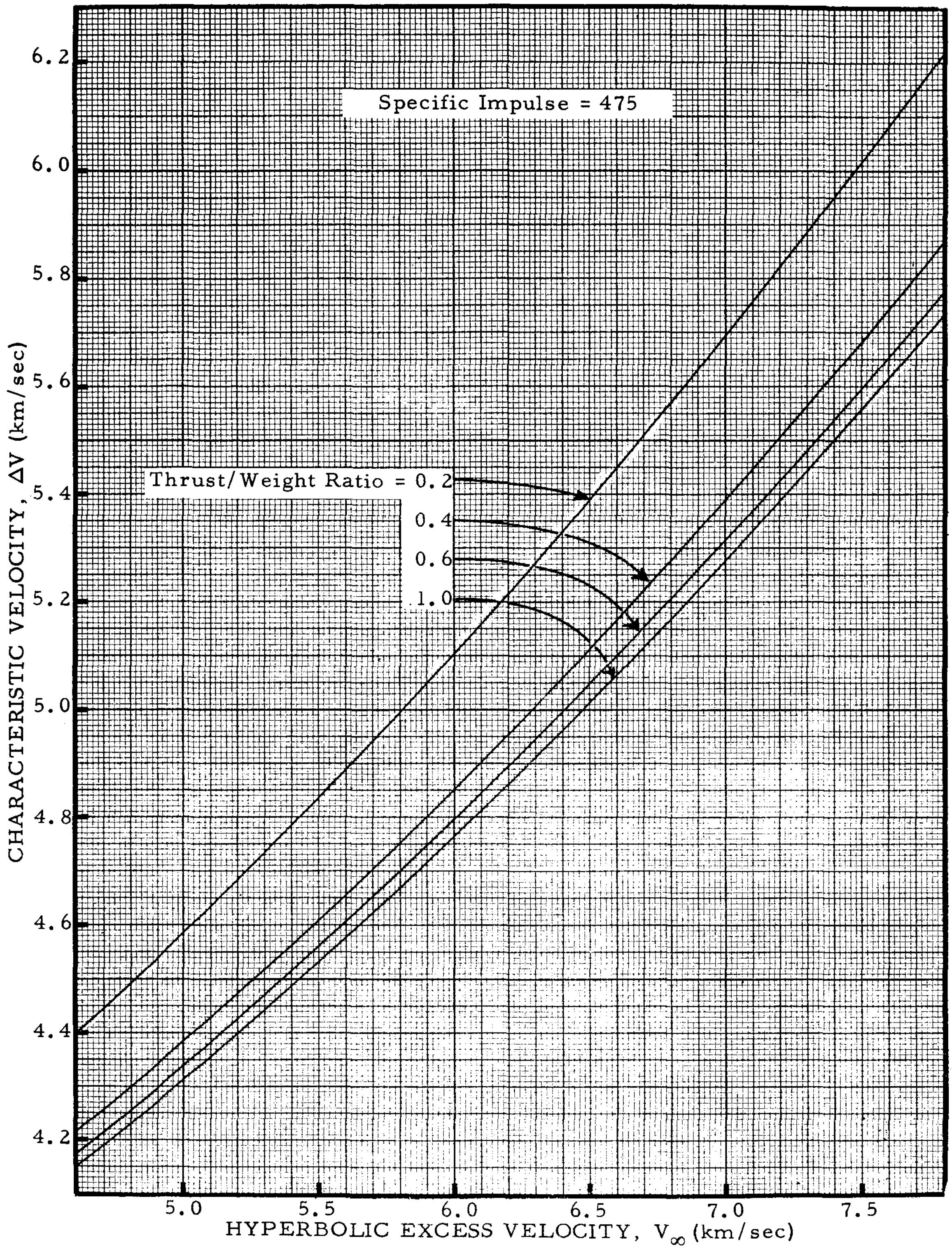


FIGURE 4b. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

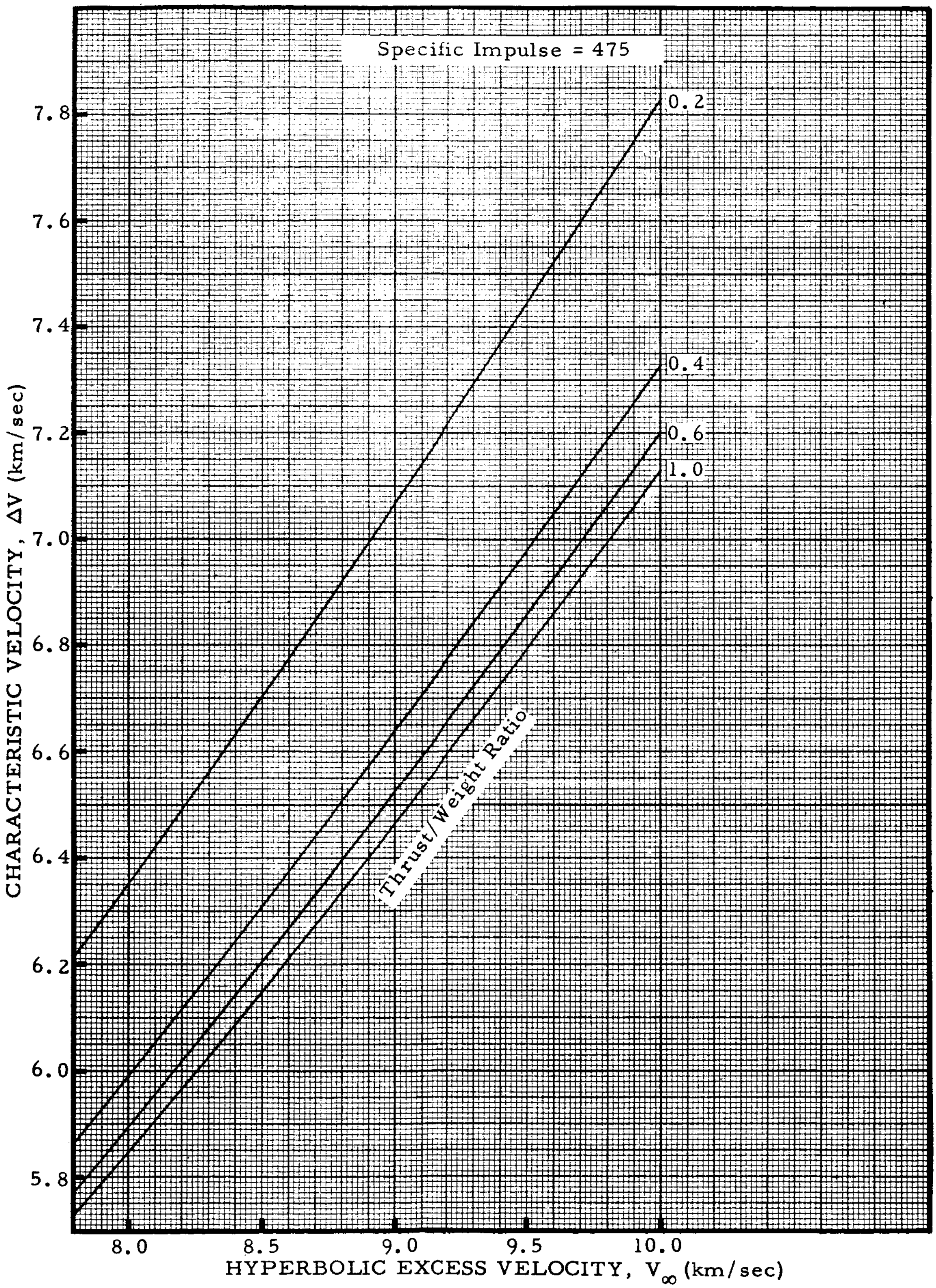


FIGURE 4c. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_{∞} (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 475 SECONDS

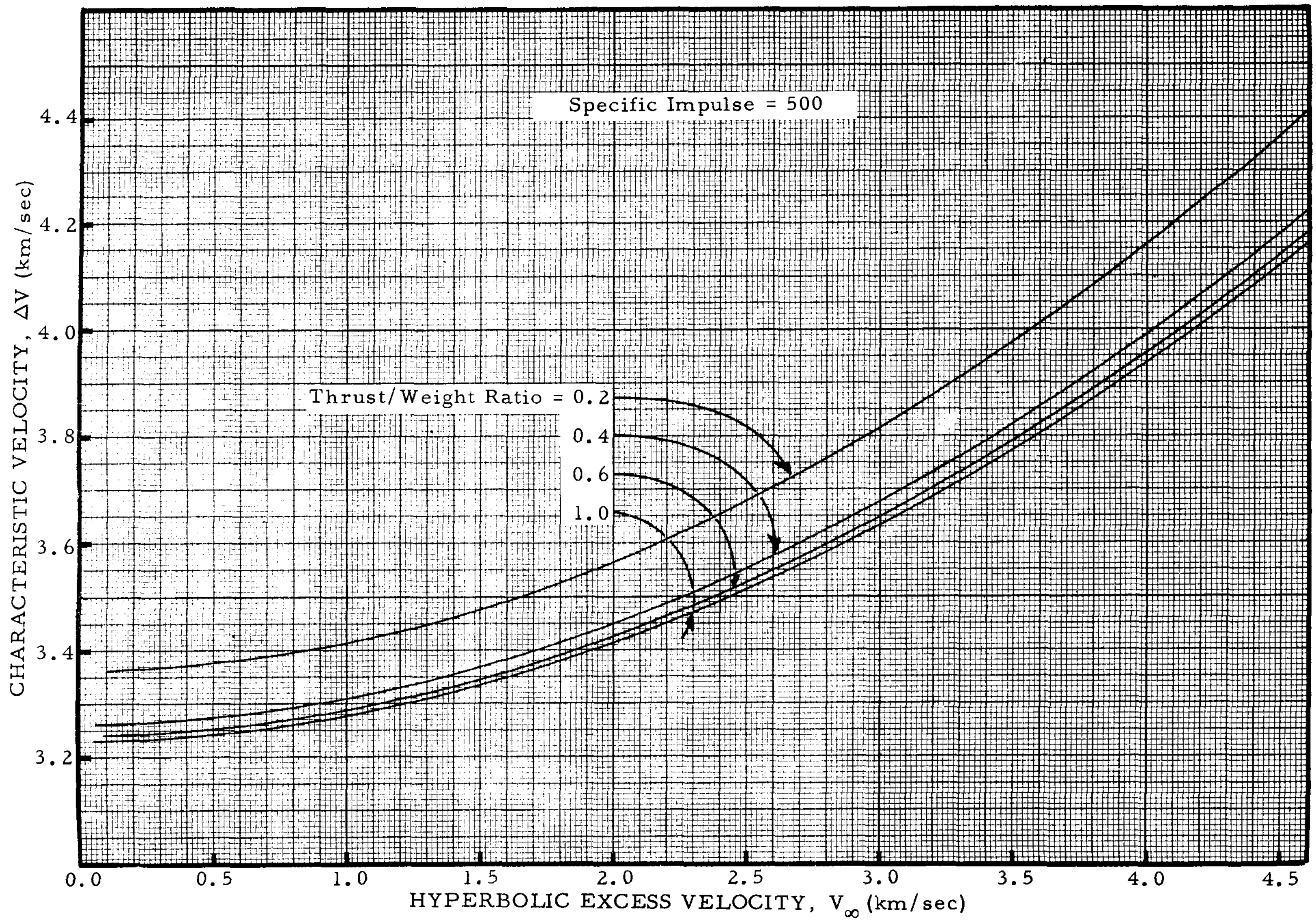


FIGURE 5a. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

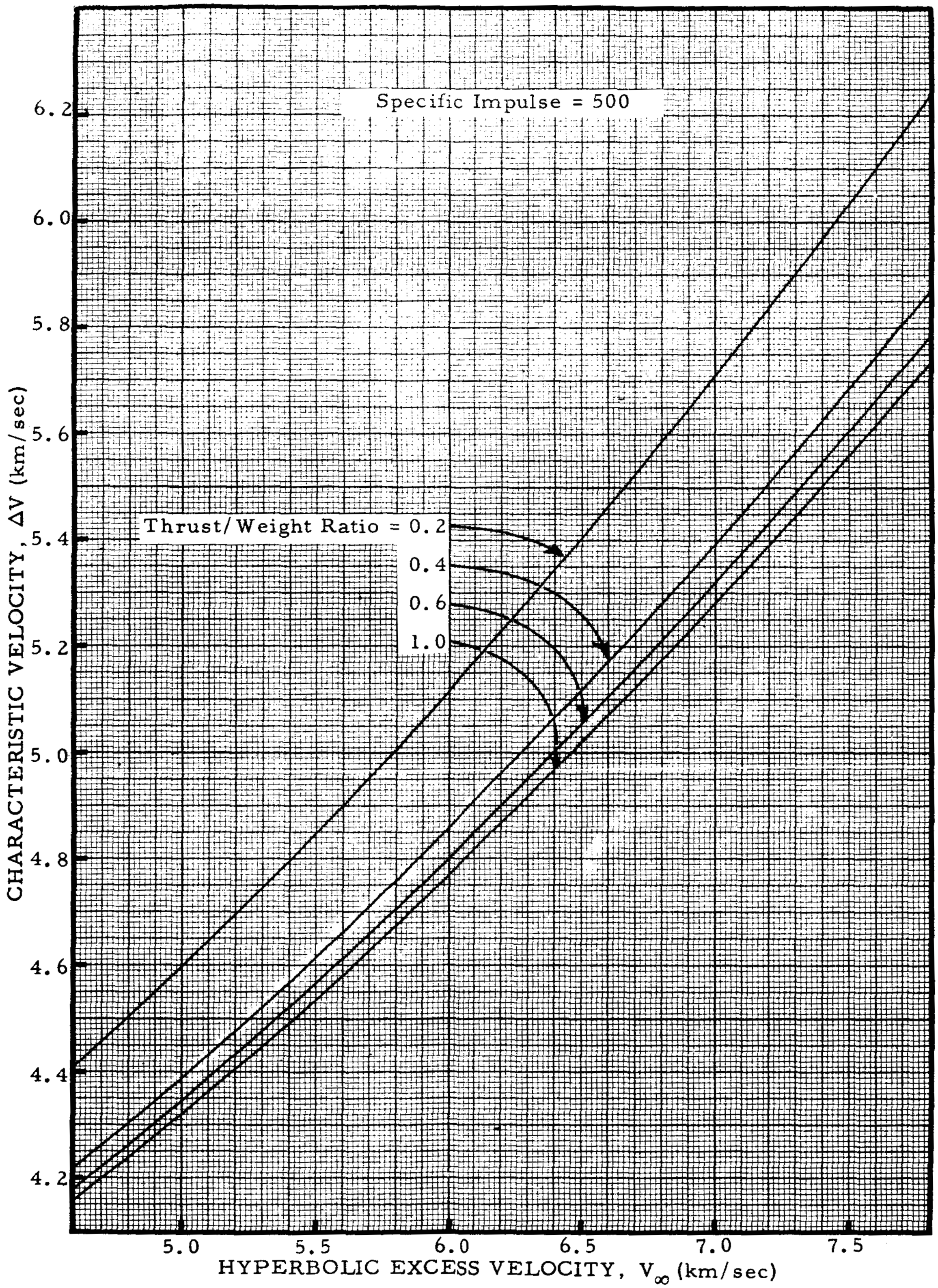


FIGURE 5b. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS

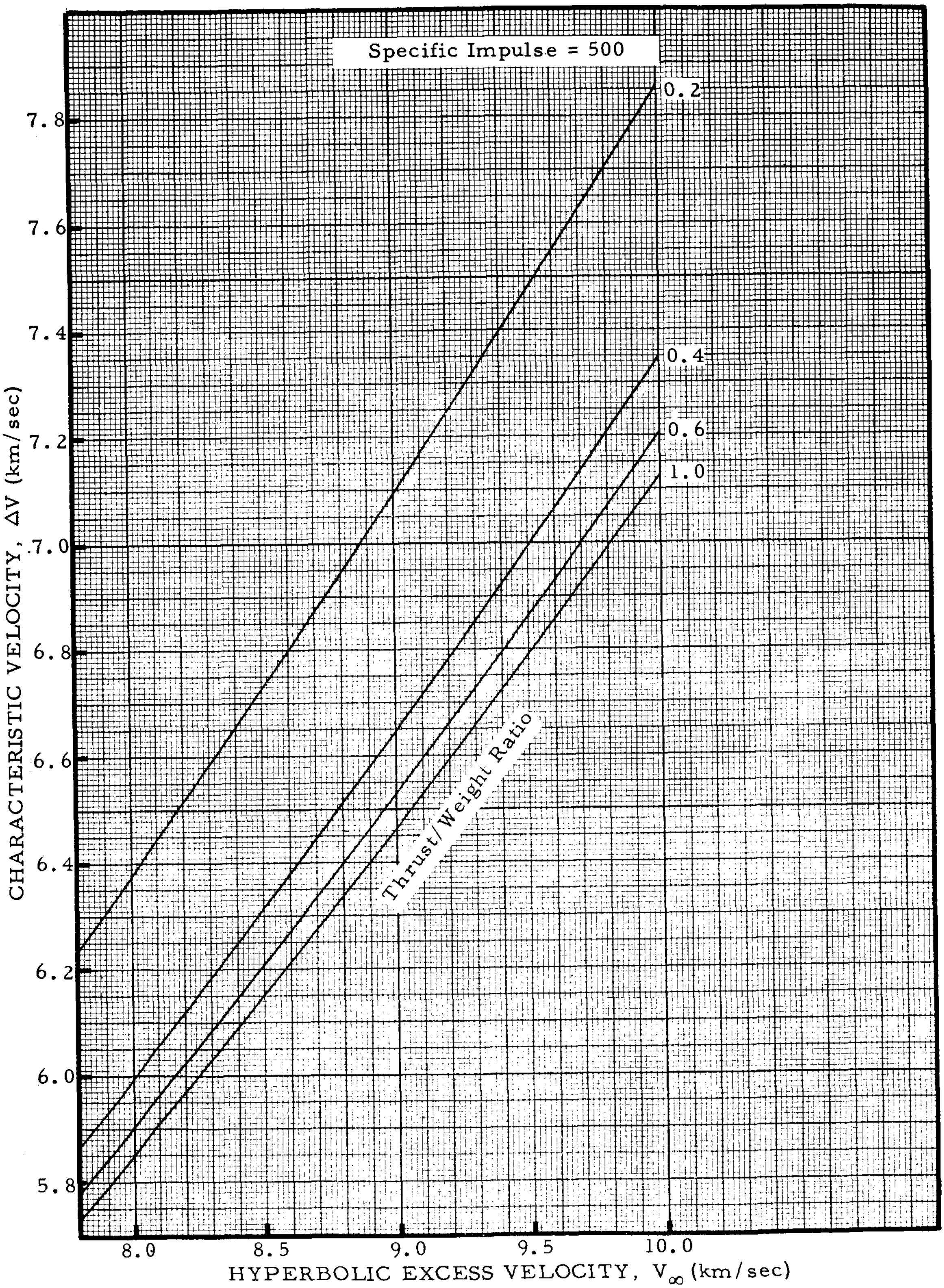
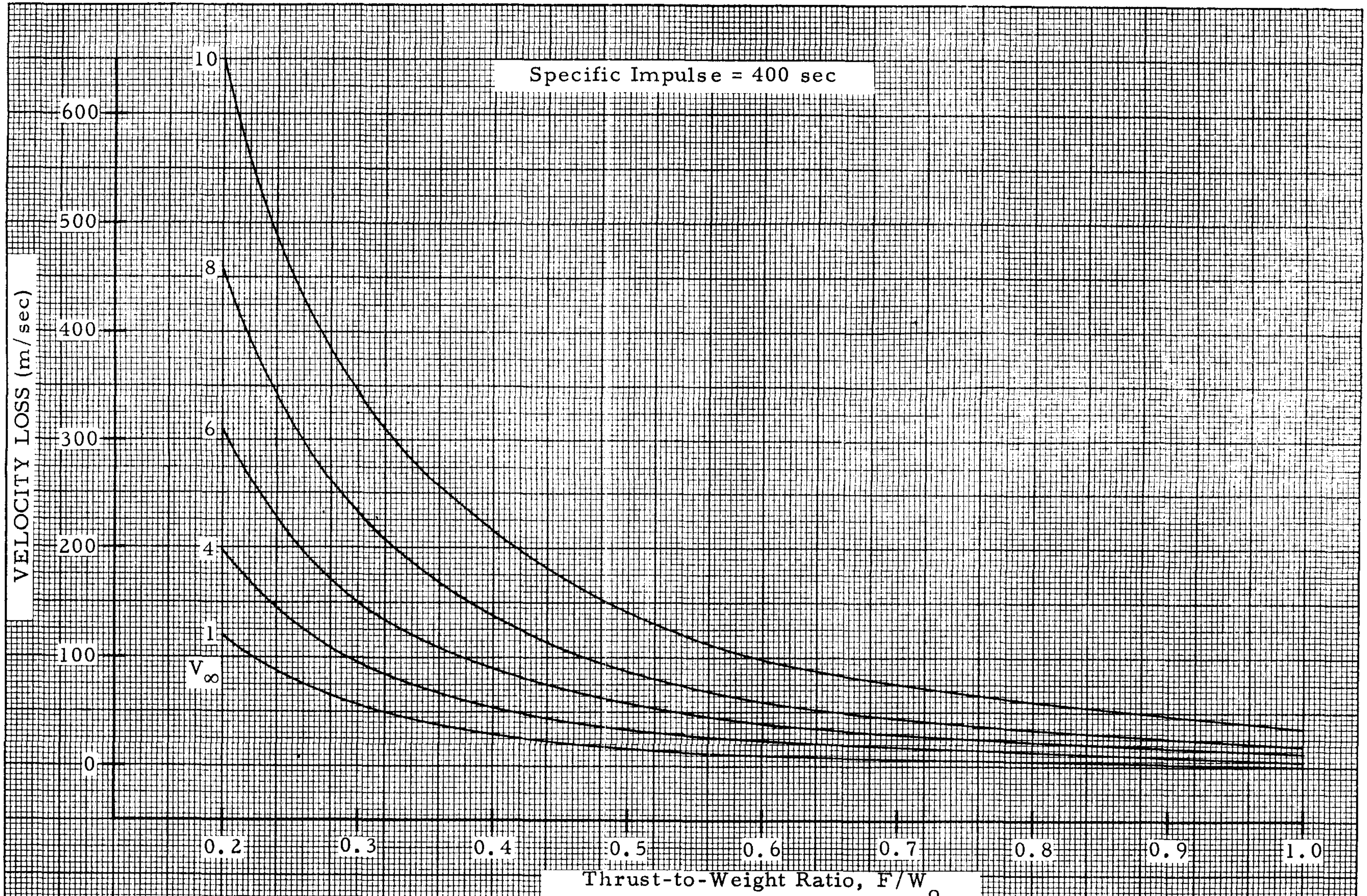


FIGURE 5c. CHARACTERISTIC VELOCITY, ΔV (km/sec), VERSUS HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 500 SECONDS



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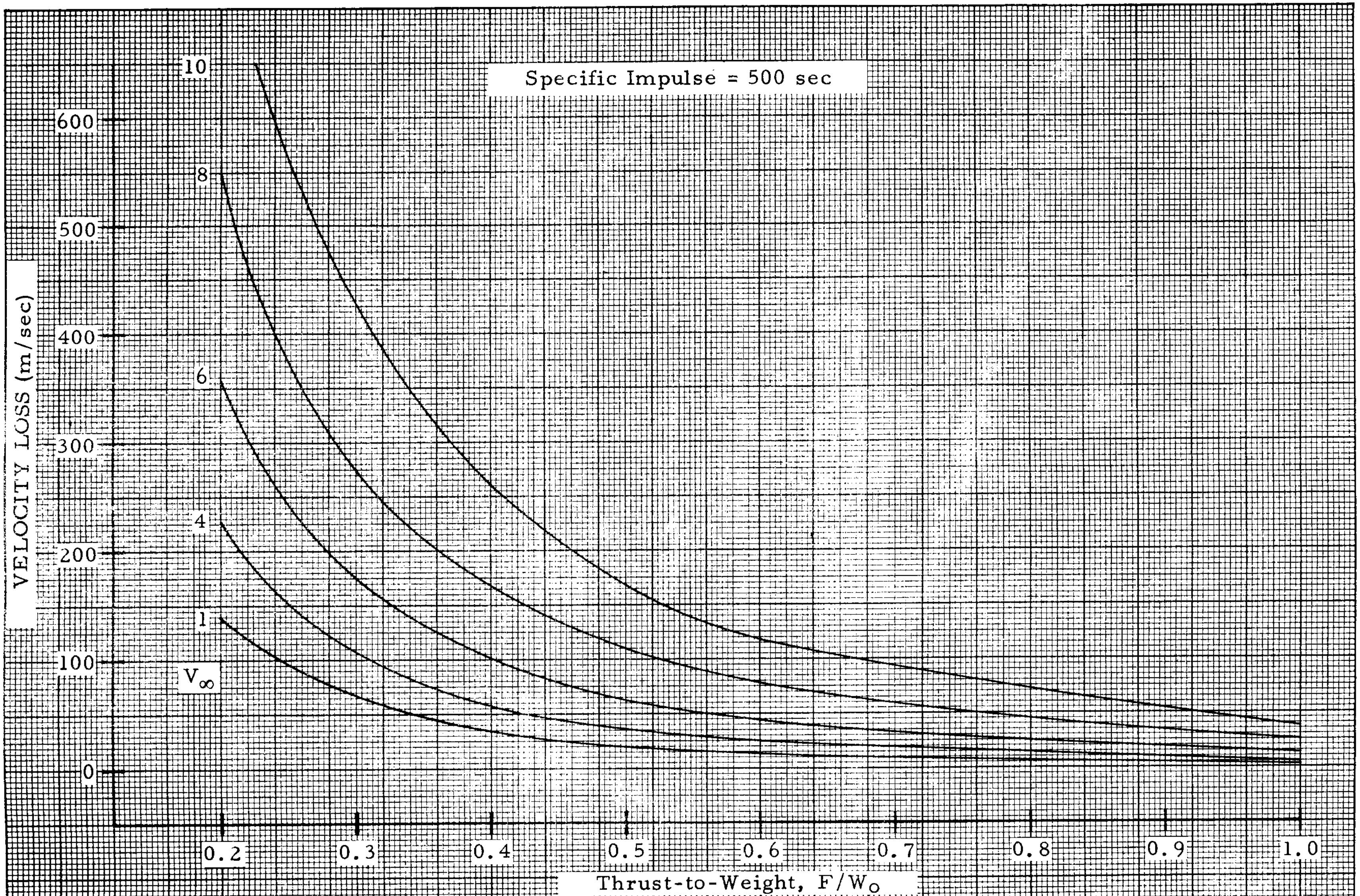

FIGURE 7. VELOCITY LOSS (m/sec) DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY, V_∞ (km/sec), AS A PARAMETER

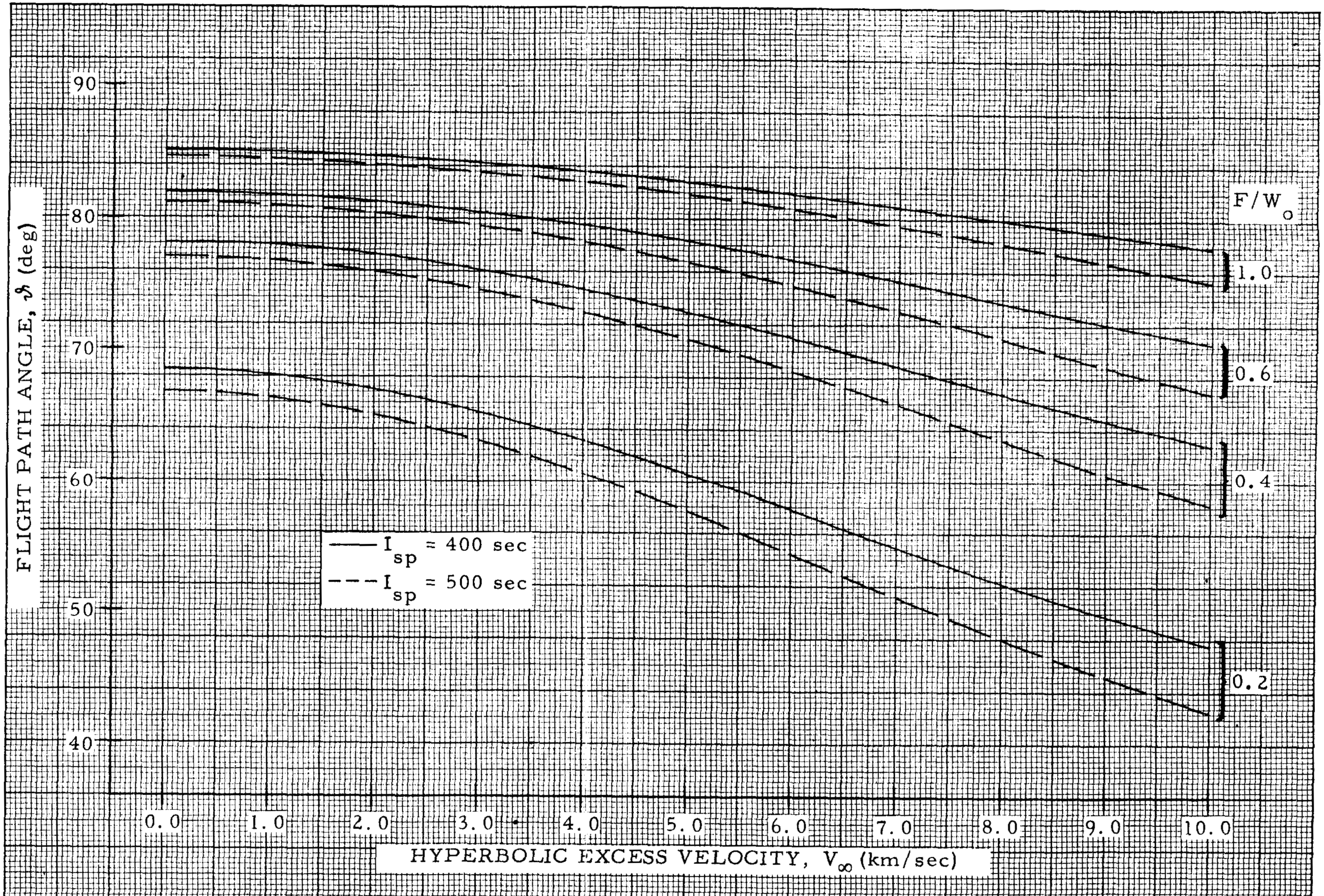

FIGURE 8. FLIGHT PATH ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

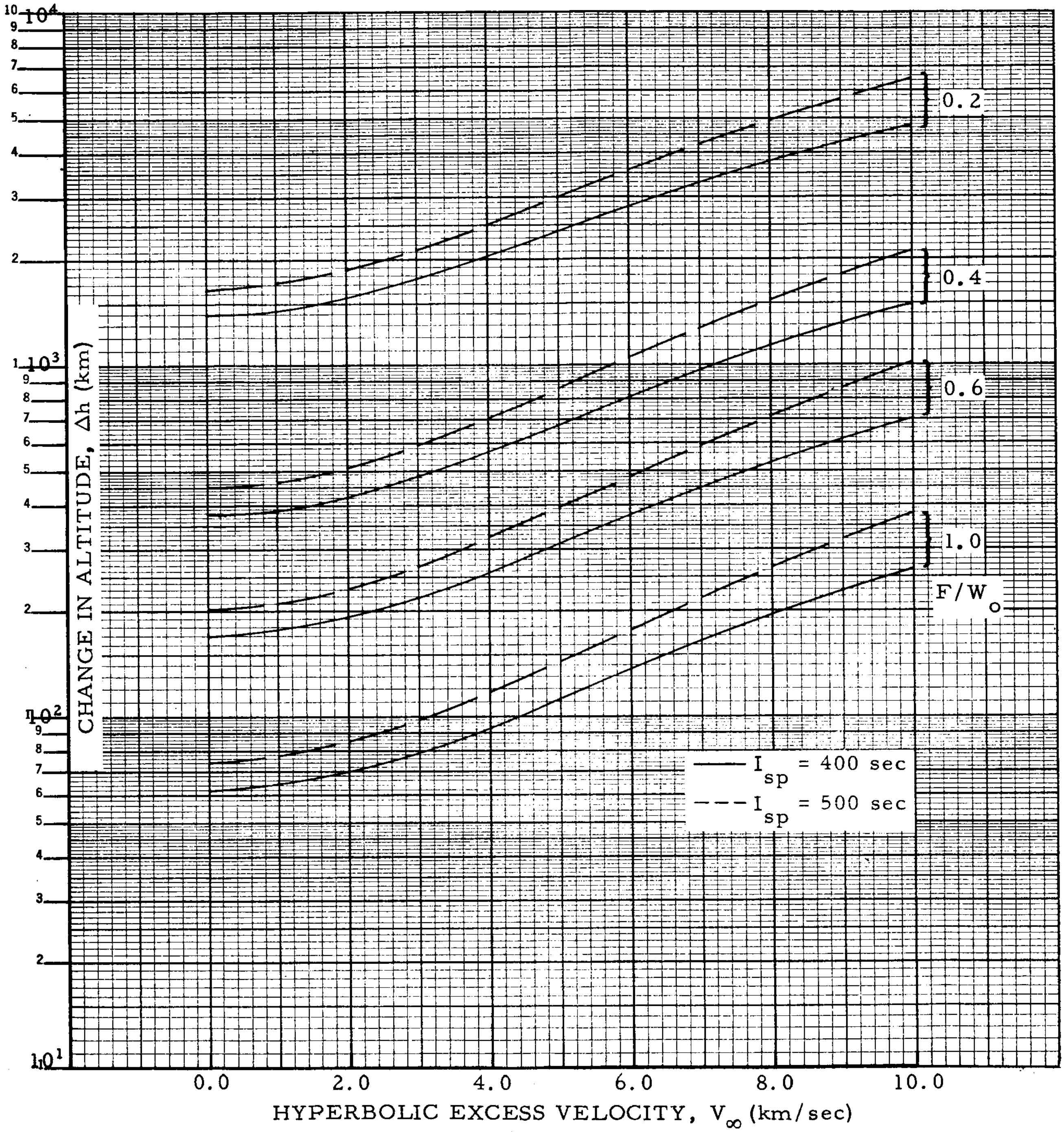

FIGURE 9. CHANGE IN ALTITUDE (km) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

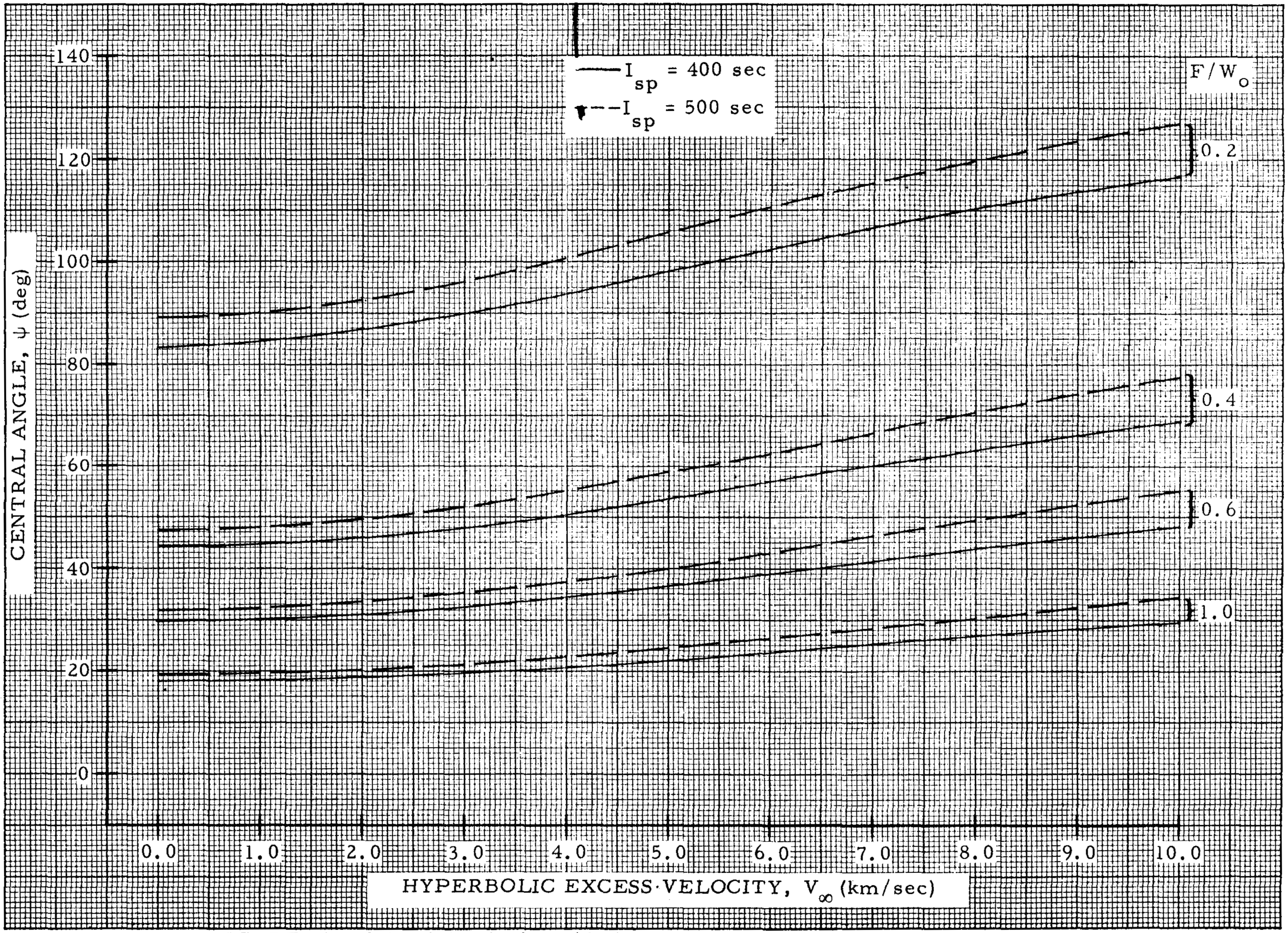

FIGURE 10. CENTRAL ANGLE (deg) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

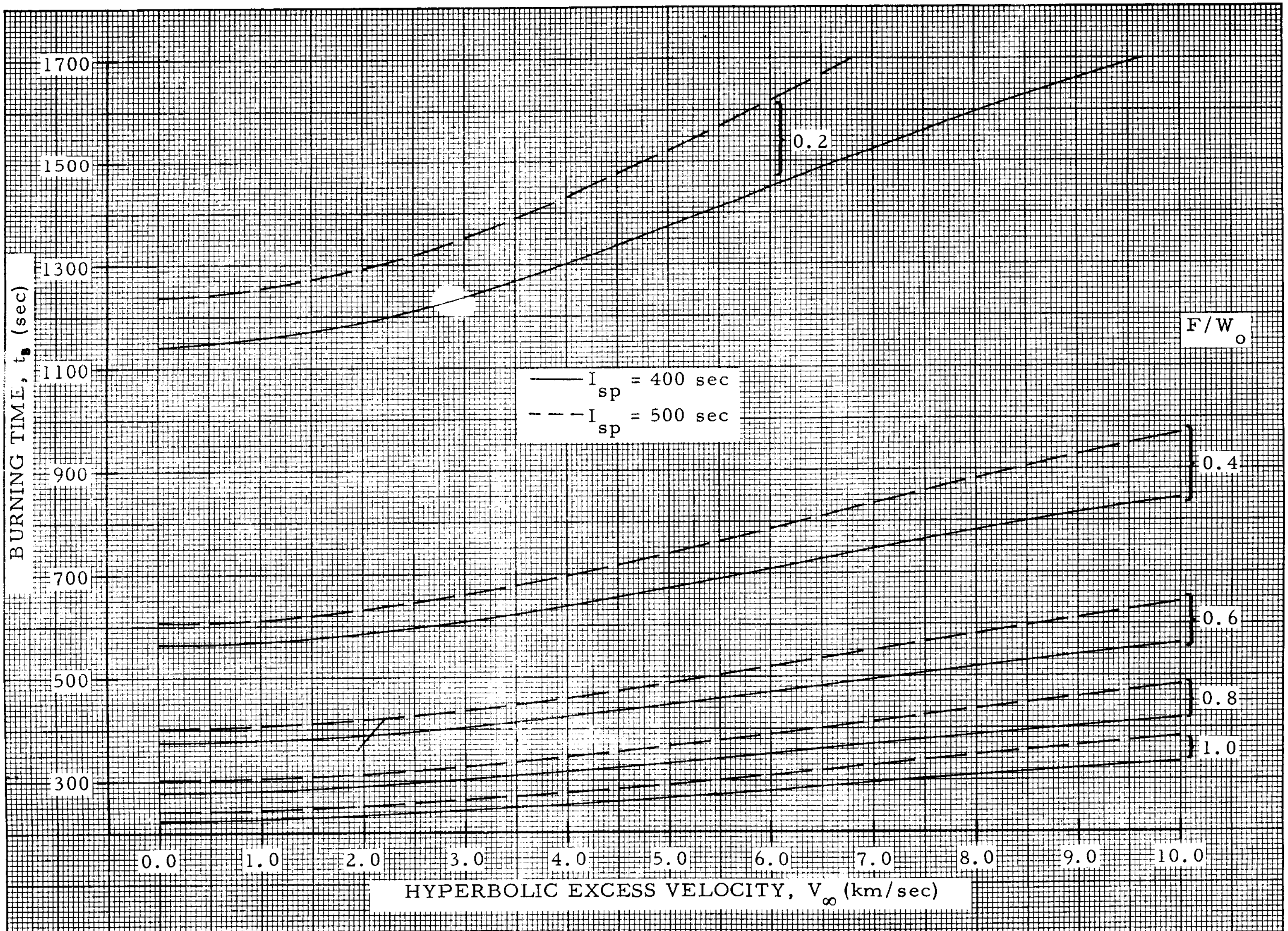

FIGURE 11. BURNING TIME (sec) VERSUS HYPERBOLIC EXCESS VELOCITY (km/sec) WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER

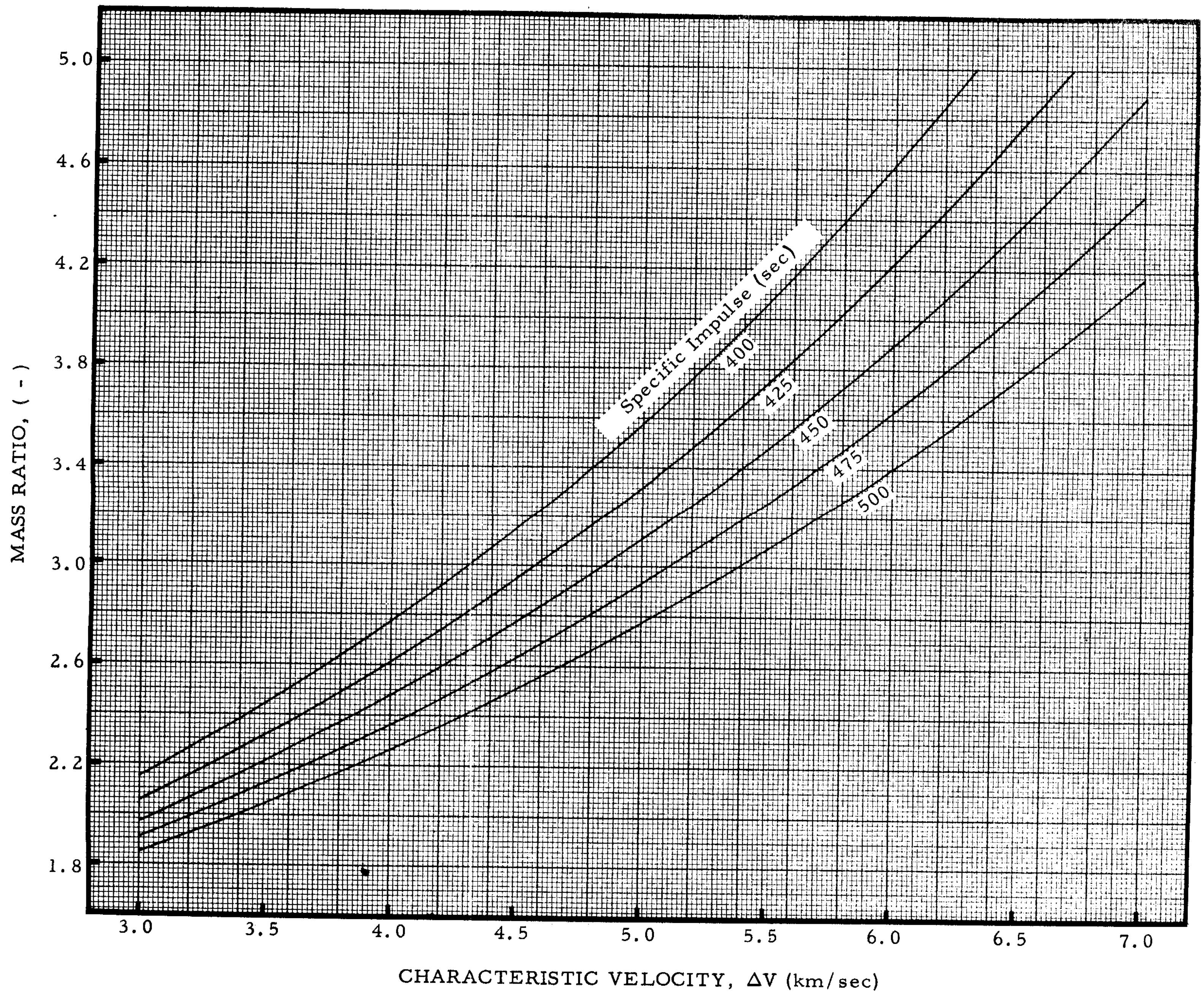

FIGURE 12. MASS RATIO VERSUS CHARACTERISTIC VELOCITY (km/sec) WITH SPECIFIC IMPULSE AS A PARAMETER

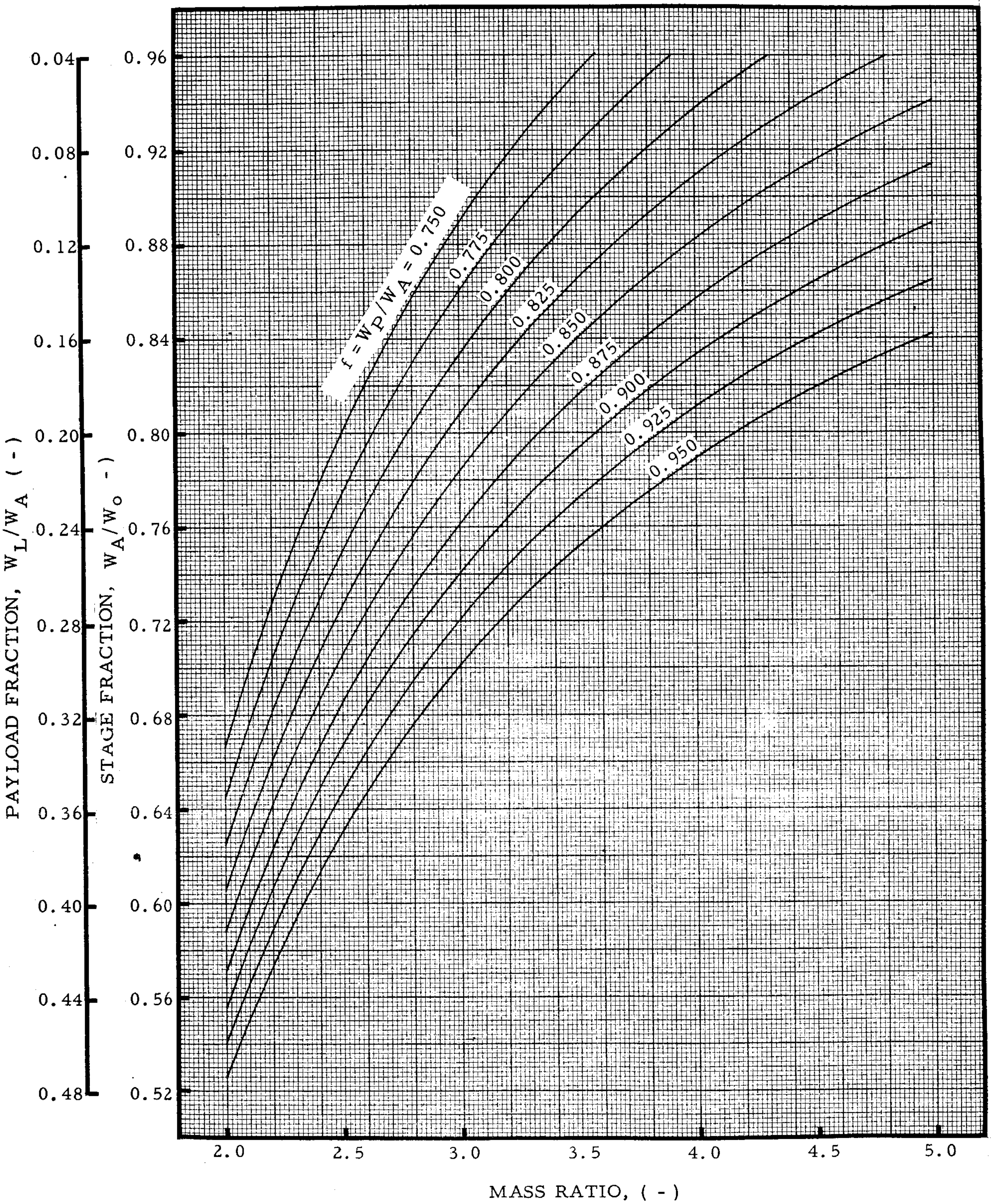

FIGURE 13. PAYLOAD FRACTION AND STAGE FRACTION VERSUS MASS RATIO WITH STAGE MASS FRACTION AS A PARAMETER

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PERFORMANCE ANALYSIS OF HIGH-ENERGY CHEMICAL STAGES FOR INTERPLANETARY MISSIONS

PART I

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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